On the Faddeev-Popov determinant in Regge calculus

V.M.Khatsymovsky

Budker Institute of Nuclear Physics

Novosibirsk, 630090, Russia

E-mail address: khatsym@inp.nsk.su

Abstract

The functional integral measure in the 4D Regge calculus normalised w.r.t. the DeWitt supermetric on the space of metrics is considered. The Faddeev-Popov factor in the measure is shown according to the previous author's work on the continuous fields in Regge calculus to be generally ill-defined due to the conical singularities. Possible resolution of this problem is discretisation of the gravity ghost (gauge) field by, e.g., confining ourselves to the affine transformations of the affine frames in the simplices. This results in the singularity of the functional measure in the vicinity of the flat background, where part of the physical degrees of freedom connected with linklengths become gauge ones.

The functional integral approach to quantisation of Regge calculus based on diffeo-morphisms has been started probably in ref. [1] and further developed by Menotti and Peirano in a number of papers, refs. [2, 3]. One of basic ingredients of this approach is Faddeev-Popov determinant which represents the gauge volume of geometries over which functional integration is performed. It is determinant of some second order differential operator acting on the space tangential to the diffeomorphism group, and the elements of this space, vectors of infinitesimal frame deformations, thus play the role of ghost fields.

Logarithm of the Faddeev-Popov factor gives, in fact, effective action of the ghost field. This field is continuous one. Meanwhile, it was shown in the recent author's paper ref. [4] that such action can suffer from the infinities unremovable by the UV renormali-sation procedure. This can be checked by studying the conformal (trace) anomaly of this field which due to occurence of curvature bilinears and δ -function-like nature of Regge curvature turns out to contain such terms.

In the present letter we calculate trace anomaly of the (vector) Faddeev-Popov ghost field for the particular choice of free constant in the DeWitt metric, namely that used in ref. [1]. The divergent part turns out to be nonzero and, moreover, of the same sign as the divergent parts in the case of matter fields studied in ref. [4], therefore one hardly can hope that any cancellation occurs. Of course, in the case of conformally noninvariant fields such as vector ghost one, there are also contributions to the trace of stress-energy tensor depending on the state of the field, therefore our argumentation here is that the cancellation of singularities can occur only for special choice of the state of the field which is equivalent to imposing a new constraint not present in the underlying continuous theory.

Aiming at constructing finite anzats for the measure we propose to confine ourselves by a discrete number of gauge degrees of freedom corresponding to the affine transforma-tions of the affine frames inside the simplices. If we adopt this, the measure is singular in the vicinity of the flat background. This is because gauge degrees of freedom are absorbed in this background by the physical ones (linklength variations) and Gaussian normalisation integral for the measure diverges.

Let us begin with some definitions of refs. [1, 2, 3] of interest. The DeWitt superme-tric on the infinitesimal deformations of metric δg_{ik} is chosen in the form [1]

$$(\delta g, \delta g) = \int dx G^{iklm} \delta g_{ik} \delta g_{lm} \tag{1}$$

with

$$G^{iklm} = \frac{1}{2} \sqrt{g} (g^{il} g^{km} + g^{im} g^{kl} - g^{ik} g^{lm}). \tag{2}$$

For the infinitesimal coordinate shifts ξ_μ (ghost field) and gauge metric deformations $\delta g_{ik} = \nabla_i \xi_k + \nabla_k \xi_i$ Gaussian integral normalisation of the measure leads, in particular, to computation of the determinant of some second order differential operator so that we need to compute effective action for the field ξ with Lagrangian density proportional to

$$\xi_i (\delta^i_k \nabla^2 + R^i_k) \xi^k \tag{3}$$

This operator differs by the sign at R_k^i from the analogous one in the electromagnetism, and we calculate the trace anomaly for it using asymptotic expansion of $< x| \exp{(tA)}|x >$ [5, 6, 7] where operator A just appears in (3). The trace anomaly of interest is given by t^{-n} term at $n = 0$. The asymptotic expansion itself can be found representing A as the sum of electromagnetic operator B (differing from A by the sign of R_k^i) and $2\mathcal{R}$, the $\mathcal{R}$ denoting matrix R_k^i, and by disentangling exponents. The explicit expressions look like

$$\exp{(tA)} = \{1 + 2\mathcal{R}t + t^2(2\mathcal{R}^2 + [B, \mathcal{R}]) + O(t^3)\} \exp{(tB)} \tag{4}$$

and

$$
\begin{aligned}
< x| \exp{(tB)}|x >_k^i &= \frac{1}{(4\pi t)^2} \left\{ \delta_k^i + t\left(\frac{1}{6}R\delta_k^i - R_k^i\right) + t^2\left[\delta_k^i\left(\frac{1}{72}R^2\right.\right.\right. \\
&\quad - \frac{1}{180}R_{lm}R^{lm} + \frac{1}{180}R_{lmpq}R^{lmpq} - \frac{1}{30}\Delta R\left.\right) + \frac{1}{15}R_m^l R_{kl}^{im} \\
&\quad + \frac{13}{30}R_{lk}R^{li} - \frac{1}{6}RR_k^i - \frac{1}{12}R_{kl}^{mp}R_{mp}^{il} + \frac{1}{6}\Delta R_k^i\left.\right]\right\} + ... \tag{5}
\end{aligned}
$$

One should be careful when calculating trace of the product of the differential operators and $\exp{(tB)}$ in the RHS of (4) and use for this purpose the $< y| \exp{(tB)}|x >$ differing from (5) by appearance, in the required orders, of the heat kernel exponent

$$\exp\left[-\frac{(y-x)^2}{4t}\right]$$

in the RHS of (5). It is useful to write out the result for the trace anomaly for the operator $B + \epsilon\mathcal{R}$,

$$
\begin{aligned}
< T_\mu^\mu > &= \frac{1}{(4\pi)^2}\left[\frac{1}{12}\left(\epsilon - \frac{2}{3}\right)R^2 + \frac{1}{12}\left(\epsilon + \frac{1}{5}\right)\Delta R + \left(\frac{1}{4}\epsilon^2 - \frac{1}{2}\epsilon + \frac{43}{180}\right)R_{ik}R^{ik}\right. \\
&\quad - \left.\frac{11}{360}R_{iklm}R^{iklm}\right]. \tag{6}
\end{aligned}
$$

At $\epsilon = 0$ we reproduce the trace anomaly for the electromagnetic operator (not for the electromagnetic field itself: for this field one should also subtract scalar ghost contribution); at $\epsilon = 2$ we have this for the vector ghost operator of interest.

According to the previous author's paper [4] the curvature bilinears are ill-defined on the 4D Regge lattice and have unremovable by the UV renormalisation infinite parts; substituting the latter parts into the $< T_\mu^\mu >$ we find that the whole singular part is proportional to

$$\frac{1}{4}\epsilon^2 - \frac{1}{3}\epsilon + \frac{1}{15}. \tag{7}$$

This is 1/15 and 2/5 for the electromagnetic and the considered ghost operator, respectively; in this scale the electromagnetic field itself gives 1/30, i.e. of the same sign. As it was found in ref. [4], contributions from the other matter fields have the same sign.

This means that it is not possible to have situation with the mutual cancellation of the ill-defined contributions from the different fields.

The singularity of the Faddeev-Popov factor in the measure makes us to reduce the continuum number of the gauge degrees of freedom to the discrete number; in fact, the only natural choice on the Regge lattice is to introduce gauge field as that describing infinitesimal changes ξ_a^i of the coordinates x_a^i of the vertices a. The coordinate frame is piecewise-affine and uniquely defined inside each 4-simplex σ by the coordinates of it's vertices. We are interested in the change of metric δ_{ik} due to variations of both vertex coordinates and the lengths squared $s_{(ab)}$ of the links connecting vertices a and b. The supermetric form (1) on this change can be most conveniently written using the so-called edge components $\delta g_{(ab)}$ of the symmetric second rank tensor [8], here δg_{ik},

$$\delta g_{(ab)} \equiv l_{ab}^i l_{ab}^k \delta g_{ik} \tag{8}$$

where $l_{ab}^i \equiv x_a^i - x_b^i$. In terms of length and coordinate variations these can be found in a simplex σ to be

$$\delta g_{\sigma(ab)} = \delta s_{(ab)} - 2 g_{\sigma ik} l_{ab}^i (\xi_a^k - \xi_b^k). \tag{9}$$

Here $g_{\sigma ik}$ is the metric inside σ. With these notations according to ref. [8] we can find

$$(\delta g, \delta g) = \sum_\sigma \sum_{(ab),(cd) \subset \sigma} \left[\frac{1}{V_\sigma^4} \frac{\partial V_\sigma^2}{\partial s_{(ab)}} \frac{\partial V_\sigma^2}{\partial s_{(cd)}} - \frac{2}{V_\sigma^2} \frac{\partial^2 V_\sigma^2}{\partial s_{(ab)} \partial s_{(cd)}} \right] \delta g_{\sigma(ab)} \delta g_{\sigma(cd)}, \tag{10}$$

V_σ being the volume of the simplex. When normalising the measure w.r.t. the DeWitt metric we perform (Gaussian) integration of $\exp(-(\delta g, \delta g))$ over $d\xi$, $d\delta s$.

If Regge spacetime is flat, the metric $g_{\sigma ik}$ entering (9) can be chosen the same for all the simplices σ. Therefore $\xi_a^k - \xi_b^k$ can be absorbed by $\delta s_{(ab)}$. This means that (generally convergent) Gaussian integral becomes divergent since the form $(\delta g, \delta g)$ becomes degenerate.

Singularity of the measure suggested in the completely discrete Regge spacetime in the vicinity of the flat background has a parallel with the author's paper [9] where the continuous time (3+1)D Regge calculus has been formulated in the canonical form. In the latter case the functional integral constructed according to the Dirac prescription turns out to be singular in the vicinity of the flat background too, and the reason for that is connected with converting a part of the second class constraints into the first class ones, i. e. again has the symmetry origin.

The possibility which still remains to be studied is using the form of the DeWitt metric more general than (2) where the coefficient at the term $g^{ik} g^{lm}$ is arbitrary constant as in [2, 3]. The possibility a'priory exists that this coefficient can be chosen to make singular part in the trace anomaly vanishing. Of course, this does not necessarily mean that the effective action is well-defined, but admit such possibility. In this case the operator A is the sum of not only Laplacian and curvature term, but also of the term $\sim \nabla^i \nabla_k$. This term makes disentangling the exponent like (4) much more difficult (infinite number of terms is required there) and, probably, expansion around the electromagnetic case does not give any advantage. Recently corresponding calculations are in progress.

References

[1] Jevicki A and Ninomiya M 1986 *Phys.Rev.D* **33** 1634.

[2] Menotti P and Peirano P P 1995 *Phys.Lett.* **353B** 444;
1996 *Nucl.Phys.B(Proc.Suppl.)* **47** 633; 1996 *Nucl.Phys.B* **473** 426.

[3] Menotti P and Peirano P P 1997 *Nucl.Phys.B* **488** 719.

[4] Khatsymovsky V *Continuous matter fields in Regge calculus*, 2000, gr-qc/0012095.

[5] Schwarz A S 1978 *Lett.Math.Phys.* **2** 247; 1979 *Commun.Math.Phys.* **64** 233.

[6] Christensen S M and Duff M J 1978 *Phys.Lett.* **76B** 571.

[7] Christensen S M 1978 *Phys.Rev.D* **17** 946.

[8] Piran T and Williams R M 1986 *Phys.Rev.D* **33** 1622.

[9] Khatsymovsky V M 1995 *Gen.Rel.Grav.* **27** 583.